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ON THE PARTIAL DERIVATIVES OF THE POTENTIAL FUNCTION IN THE PROBLEM OF n BODIES.

By MR. W. J. HUSSEY, Ann Arbor, Mich.

Denoting the masses of the bodies by m_1, m_2, ... m_n, and their distances apart by r_{12}, r_{13}, ... $r_{(n-1)n}$, the potential function is

$$V = k^2 \left[\frac{m_1 m_2}{r_{12}} + \frac{m_1 m_3}{r_{13}} + \ldots + \frac{m_{n-1} m_n}{r_{(n-1)n}} \right],$$

in which k^2 is a constant.

Since V is a function of the distances, it is a function of the differences of the co-ordinates, and consequently the sum of its partial derivatives with respect to each co-ordinate is equal to zero. Therefore

$$\frac{\partial V}{\partial x_1} + \frac{\partial V}{\partial x_2} + \ldots + \frac{\partial V}{\partial x_n} = 0,$$

and similar expressions in y and z. These may be written

$$\left.\begin{aligned} \Sigma \frac{\partial V}{\partial x} &= 0, \\ \Sigma \frac{\partial V}{\partial y} &= 0, \\ \Sigma \frac{\partial V}{\partial z} &= 0. \end{aligned}\right\} \qquad (1)$$

Further, if the axes of co-ordinates are changed by a rotation about the axis of x, through an angle φ, y and z become

$$y = y' \cos \varphi - z' \sin \varphi,$$

$$z = y' \sin \varphi + z' \cos \varphi.$$

By this transformation V remains unchanged and independent of φ, consequently its partial derivative with respect to φ is equal to zero. This gives

$$\frac{\partial V}{\partial y_1}\frac{dy_1}{d\varphi} + \frac{\partial V}{\partial z_1}\frac{dz_1}{d\varphi} + \ldots = 0\,,$$

$$- z_1\frac{\partial V}{\partial y_1} + y_1\frac{\partial V}{\partial z_1} - \ldots = 0\,.$$

Or

$$\Sigma\left[y\frac{\partial V}{\partial z} - z\frac{\partial V}{\partial y}\right] = 0,$$

and similarly,

$$\left.\begin{aligned} &\Sigma\left[y\frac{\partial V}{\partial z} - z\frac{\partial V}{\partial y}\right] = 0,\\ &\Sigma\left[z\frac{\partial V}{\partial x} - x\frac{\partial V}{\partial z}\right] = 0,\\ &\Sigma\left[x\frac{\partial V}{\partial y} - y\frac{\partial V}{\partial x}\right] = 0.\end{aligned}\right\} \qquad (2)$$

Equations (1) and (2) are of fundamental importance in the problem of n bodies, and this derivation of them is the most direct with which I am acquainted.